Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A is a null matrix then **[1]**

a) A is a square matrix

b) A is a cube matrix

c) A is not a square matrix

d) both A is a square matrix and A is not a square matrix

2. If A is a 3-rowed square matrix and |3A| = k |A| then k = ? **[1]**

a) 9

b) 27

c) 3

d) 1

3. The solution of the differential equation $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$, is **[1]**

a) $\tan^{-1}\left(\frac{y}{x}\right) = \log y + C$

b) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

c) $\tan^{-1}\left(\frac{x}{y}\right) = \log y + C$

d) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$

4. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is **[1]**

a) discontinuous at exactly three points

b) discontinuous at only one point

c) discontinuous at exactly four points

d) discontinuous at exactly two points

5. The projections of a line segment on X, Y and Z axes are 12, 4 and 3 respectively. The length and direction cosines of the line segment are **[1]**

a) $11; \frac{12}{11}, \frac{14}{11}, \frac{3}{11}$

b) $19; \frac{12}{19}, \frac{4}{19}, \frac{3}{19}$

c) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

d) $15; \frac{12}{15}, \frac{14}{15}, \frac{3}{15}$

[1]

6. Degree of the differential equation sin x + $\cos\left(\frac{dy}{dx}\right) = y^2$ is

a) 2
b) 0
c) 1
d) not defined

7. The common region determined by all the constraints of a linear programming problem is called: **[1]**

a) a feasible region
b) a bounded region
c) an optimal region
d) an unbounded region

8. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to **[1]**

a) $\frac{1}{4}$
b) 1
c) $\frac{1}{3}$
d) $\frac{1}{2}$

9. $\int \frac{1}{\sqrt[3]{x}} dx = ?$ **[1]**

a) $\frac{3}{2x^{\frac{2}{3}}} + C$
b) $\frac{2}{3}x^{\frac{3}{2}} + C$
c) $\frac{3}{2}x^{\frac{2}{3}} + C$
d) $\frac{2}{3x^{\frac{2}{3}}} + C$

10. If for the matrix A = $\begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, $A + A' = 2\sqrt{3}I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is: **[1]**

a) 0
b) $\frac{\pi}{6}$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{3}$

11. The objective function Z = 4 x + 3 y can be maximised subjected to the constraints 3x + 4y $\leq$ 24, 8x + 6y $\leq$ 48 , x $\leq$ 5 , y $\leq$ 6 ; x, y $\geq$ 0 **[1]**

a) at a finite number of points
b) at only one point
c) at an infinite number of points
d) at two points only

12. Let f(x) = |x| and g(x) = $|x^3|$, then **[1]**

a) f(x) and g(x) both are continuous at x = 0
b) f(x) is differentiable but g(x) is not differentiable at x = 0
c) f(x) and g(x) both are not differentiable at x = 0
d) f(x) and g(x) both are differentiable at x = 0

13. A line passes through the point A (5, -2. 4) and it is parallel to the vector $(2\hat{i} - \hat{j} + 3\hat{k})$. The vector equation of the line is **[1]**

a) $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 4\hat{k}) = \sqrt{12}$
b) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(5\hat{i} - 2\hat{j} + 4\hat{k})$
c) $\vec{r} \cdot (5\hat{i} - 2\hat{j} + 4\hat{k}) = \sqrt{14}$
d) $\vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

14. For any two events A and B, if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ equals: **[1]**

a) $\frac{8}{9}$
b) $\frac{1}{4}$
c) $\frac{5}{8}$
d) $\frac{3}{8}$

15. Which of the following is a second order differential equation? **[1]**

a) $(y')^2 + x = y^2$
b) y′y′′ + y = sin x

c) $y'' + (y')^2 + y = 0$ d) $y' = y^2$

16. The function $f(x) = x^3 + 3x$ is increasing in interval **[1]**

a) $(-\infty, 0)$ b) (0, 1)

c) $(0, \infty)$ d) $\mathbb{R}$

17. If $x = at, y = \frac{a}{t}$, then $\frac{dy}{dx}$ is: **[1]**

a) $\frac{1}{t^2}$ b) t^2

c) $-\frac{1}{t^2}$ d) $-t^2$

18. If the direction ratios of a line are 2, 3 and -6, then direction cosines of the line making obtuse angle with Y-axis are **[1]**

a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ b) $\frac{-2}{7}, \frac{3}{7}, \frac{-6}{7}$

c) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$ d) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$

19. **Assertion (A):** Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes. **[1]**

Reason (R): The acute angle between the lines x - 2 = 0 and $\sqrt{3x - y - 2}$ is 30^o.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** $\int xe^{x^2} dx = \frac{1}{2}e^{x^2} + c$ **[1]**

Reason (R): To solve above integral put $x^2 = t$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Find equation of line joining (3, 1) and (9, 3) using determinants. **[2]**

OR

Evaluate the determinant $\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$ by expanding it along first column.

22. Evaluate $\int_1^3 \frac{\cos(\log x)}{x} dx$ **[2]**

23. The volume of a cube is increasing at a rate of 8 cm^3/ sec . How fast is the surface area increasing when the length of an edge is 12 cm? **[2]**

OR

Find the intervals in which $f(x) = (x + 2)e^{-x}$ is increasing or decreasing.

24. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event **number is even** and B be the event **number is marked red**. Find whether the events A and B are independent or not. **[2]**

25. Find the sum of the order and the degree of the differential equation: $\left(x + \frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$ **[2]**

Section C

26. Find: $\int \frac{x^3+1}{x^3-x} dx$ **[3]**

27. For what value of λ, the function defined by f(x) = $\begin{cases} \lambda\left(x^2+2\right), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x>0 \end{cases}$ is continuous at x = 0? Hence, check the differentiability of f(x) at x = 0. **[3]**

28. $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x+4\sin^2 x} dx$ **[3]**

OR

Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

29. Solve the initial value problem: x dy + y dx = xy dx, y(1) = 1 **[3]**

OR

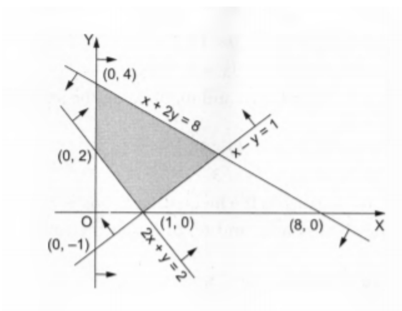
Find the particular solution of the differential equation $xe^{y/x} - y + x\frac{dy}{dx} = 0$, given that y(e) = 0.

30. Find the distance of the point with position vector $-\hat{i}-5\hat{j}-10\hat{k}$ from the point of intersection of the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$ with the plane $\overrightarrow{\mathrm{r}} \cdot (\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}})=5$ **[3]**

OR

If $\vec{a}=(\hat{i}-\hat{j}), \vec{b}=(3\hat{j}-\hat{k})$ and $\vec{c}=(7\hat{i}-\hat{k})$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d}$ = 1.

31. Find the linear constraints for which the shaded area in the figure given is the solution set. **[3]**



Section D

32. If the area bounded by the parabola $y^2=16ax$ and the line $y=4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m. **[5]**

33. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$. **[5]**

OR

If with reference to the right handed system of mutually $\perp$ unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ and $\vec{\alpha}=3\hat{i}-\hat{j}, \vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}$ is || to $\vec{\alpha}$ and $\vec{\beta}_2$ is $\perp$ to $\vec{\alpha}$

34. If $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$. **[5]**

35. A square piece of tin of side 18 cm is to be made into a box without the top, by cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume of the box. **[5]**

OR

Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

i. Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics? (1)

ii. Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics? (1)

iii. Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics? (2)

OR

Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S, D}, B = {1, 2, 3, 4, 5, 6}

i. Let $R : B \rightarrow B$ be defined by R = {(x, y): y is divisible by x}. Determine whether R is Reflexive, symmetric or transitive. (1)

ii. Raji wants to know the number of functions from A to B. How many number of functions are possible? (1)

iii. Let R be a relation on B defined by R = {(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}. Then describe R . (2)

OR

Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.): $Lf'(a) = \lim\limits_{h \to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.): $Rf'(a) = \lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

i. What is R.H.D. of f(x) at x = 1? (1)

ii. What is L.H.D. of f(x) at x = 1? (1)

iii. Check if the function f(x) is differentiable at x = 1. (2)

OR

Find f'(2) and f'(-1). (2)

Solution

Section A

1. **(a)** A is a square matrix

   Explanation:

   A square matrix is a null matrix if all its entries are zero.

2. **(b)** 27

   Explanation:

   Since the matrix is of order 3 so 3 will be taken common from each row or column.

   So, k = 27

3. **(d)** $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$

   Explanation:

   We have,

   $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

   $\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$ ...(i)

   Let y = vx

   $\frac{dy}{dx} = v + x\frac{dv}{dx}$

   $1 + v + v^2 = v + x\frac{dv}{dx}$ ....from(i)

   $1 + v^2 = x\frac{dv}{dx}$

   $\frac{dx}{x} = \frac{dv}{1+v^2}$

   $\int \frac{dx}{x} = \int \frac{dv}{1+v^2}$

   log |x| = $\tan^{-1}$x + c

4. **(a)** discontinuous at exactly three points

   Explanation:

   We have, $f(x) = \frac{4-x^2}{4x-x^3} = \frac{(4-x^2)}{x(4-x^2)}$

   $= \frac{(4-x^2)}{x(2^2-x^2)} = \frac{4-x^2}{x(2+x)(2-x)}$

   Clearly, f(x) is discontinuous at exactly three points x = 0, x = -2 and x = 2.

5. **(c)** $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

   Explanation:

   $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

   If a line makes angles α, β and γ with the axis, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ = 1 ....(i)

   Let r be the length of the line segment. Then,

   $r\cos\alpha = 12, r\cos\beta = 4, r\cos\gamma = 3$ ....(ii)

   $\Rightarrow (r\cos\alpha)^2 + (r\cos\beta)^2 + (r\cos\gamma)^2$ = $12^2 + 4^2 + 3^2$

   $\Rightarrow r^2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)$ = 169

   $\Rightarrow r^2$ (1) = 169 [From (i)]

   $\Rightarrow r = \sqrt{169}$

   $\Rightarrow$ r = $\pm$13

   $\Rightarrow$ r = 13 (since length cannot be negative)

   Substituting r = 13 in (ii)

We get,
$\cos\alpha = \frac{12}{13}, \cos\beta = \frac{4}{13}, \cos\gamma = \frac{1}{13}$
Thus, the direction cosines of the line are
$\frac{12}{13}, \frac{4}{13}, \frac{1}{13}$

6.
(d) not defined
Explanation:
not defined

7. **(a)** a feasible region
Explanation:
a feasible region

8.
(b) 1
Explanation:
$\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$
$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

9.
(c) $\frac{3}{2}x^{\frac{2}{3}} + C$
Explanation:
Given:
$\int \frac{1}{\sqrt[3]{x}} dx$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\int \frac{1}{\sqrt[3]{x}} dx = \frac{x^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} + c$
$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$
$= \frac{3}{2}x^{\frac{2}{3}} + c$

10.
(d) $\frac{\pi}{3}$
Explanation:
Given the matrix $A = \begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, we find its transpose $A' = \begin{bmatrix} \tan x & -1 \\ 1 & \tan x \end{bmatrix}$ Setting $A + A' = 2\sqrt{3}I$ leads to $2\tan x = 2\sqrt{3}$, giving $\tan x = \sqrt{3}$. Thus, $x = \frac{\pi}{3}$.

11.
(c) at an infinite number of points
Explanation:
First, we will convert the given inequations into equations, we obtain the following equations:
3x + 4y = 24, 8x + 6y = 48, x = 5, y = 6, x = 0 and y = 0
The line 3x + 4y = 24 meets the coordinate axis at A(8, 0) and B(0, 6). Join these points to obtain the line 3x + 4y = 24 Clearly, (0, 0) satisfies the inequation 3x + 4y $\leq$ 24 . So, the region in x y -plane that contains the origin represents the solution set of the given equation. The line 8x + 6y = 48 meets the coordinate axis at C(6, 0) and D(0, 8) . Join these points to obtain the line 8x + 6y = 48 Clearly, (0, 0) satisfies the inequation 8x + 6y $\leq$ 48 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.
x = 5 is the line passing through x = 5 parallel to the Y axis.

y = 6 is the line passing through y = 6 parallel to the X axis.
The region represented by x $\geq$ 0 and y $\geq$ 0 :
since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.
These lines are drawn using a suitable scale.
The corner points of the feasible region are O(0, 0), G(5, 0), $F\left(5, \frac{4}{3}\right)$, $E\left(\frac{24}{7}, \frac{24}{7}\right)$ and B(0, 6)
The values of Z at these corner points are as follows:
Corner point : Z = 4x + 3y
O(0, 0) : $4 \times 0 + 3 \times 0 = 0$
G(5, 0) : $4 \times 5 + 3 \times 0 = 20$
$F\left(5, \frac{4}{3}\right) : 4 \times 5 + 3 \times \frac{4}{3} = 24$
$E\left(\frac{24}{7}, \frac{24}{7}\right) : 4 \times \frac{24}{7} + 3 \times \frac{24}{7} = \frac{196}{7} = 28$
B(0, 6) : $4 \times 0 + 3 \times 6 = 18$
We see that the maximum value of the objective function z is 24 which is at $F\left(5, \frac{4}{3}\right)$ and $E\left(\frac{24}{7}, \frac{24}{7}\right)$ Thus, the optimal value of Z is 24
As, we know that if an LPP has two optimal solutions, then there are an infinite number of optimal solutions. Therefore, the given objective function can be subjected at an infinite number of points.

12. **(a)** f(x) and g(x) both are continuous at x = 0
Explanation:
Given f(x) = |x| and g(x) = $|x^3|$,
$$f(x)=\begin{cases}-x, & x \leq 0 \\ x, & x>0\end{cases}$$
Checking differentiability and continuity,
LHL at x =0,
$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(0-h) = 0$$
RHL at x =0,
$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} (0+h) = 0$$
And f(0)=0
Hence, f(x) is continuous at x =0.
LHD at x =0,
$$\lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$$
$$= \lim_{h \to 0} \frac{(0-h)-(0)}{-h} = -1$$
RHD at x =0,
$$\lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0+h)-f(0)}{0+h-0}$$
$$= \lim_{h \to 0} \frac{(0+h)-(0)}{h} = 1$$
$\because$ LHD $\neq$ RHD
$\therefore$ f(x) is not differentiable at x =0.
$$g(x)=\begin{cases}-x^3, & x \leq 0 \\ x^3, & x>0\end{cases}$$
Checking differentiability and continuity,
LHL at x =0,
$$\lim_{x \to 0^-} g(x) = \lim_{h \to 0} g(0-h) = \lim_{h \to 0} -(0-h)^3 = 0$$
RHL at x =0,
$$\lim_{x \to 0^+} g(x) = \lim_{h \to 0} g(0+h) = \lim_{h \to 0} (0+h)^3 = 0$$
And g(0)=0
Hence, g(x) is continuous at x =0.

LHD at x =0,

$\lim_{x\to 0^-}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0-h)-g(0)}{0-h-0}$

$=\lim_{h\to 0}\frac{(0-h)^3-(0)}{-h}=0$

RHD at x =0,

$\lim_{x\to 0^+}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0+h)-g(0)}{0+h-0}$

$=\lim_{h\to 0}\frac{(0+h)^3-(0)}{h}=0$

∵ LHD = RHD

∴ g(x) is differentiable at x =0.

13.

(d) $\vec{r}=(5\hat{i}-2\hat{j}+4\hat{k})$ $+\lambda(2\hat{i}-\hat{j}+3\hat{k})$

Explanation:

Fixed point is $5\hat{i}-2\hat{j}+4\hat{k}$ and parallel vector is $2\hat{\imath}-\hat{\jmath}+3\hat{\mathrm{k}}$

Equation $\vec{r}=5\hat{\imath}-2\hat{\jmath}+4\hat{k}+\lambda(2\hat{\imath}-\hat{\jmath}+3\hat{k})$

14.

(c) $\frac{5}{8}$

Explanation:

$P(\bar{A})=\frac{1}{2}, P(\bar{B})=\frac{2}{3}$ and $P(A\cap B)=\frac{1}{4}$

$\Rightarrow P(A)=\frac{1}{2}; P(B)=\frac{1}{3}$

We have, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

$=\frac{7}{12}$

$P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$

$=\frac{P(\overline{A\cup B})}{P(\bar{B})}$

$=\frac{1-P(A\cup B)}{P(\bar{B})}$

$=\frac{1-\frac{7}{12}}{\frac{2}{3}}$

$=\frac{5}{8}$

15.

(c) $y'' + (y')^2 + y = 0$

Explanation:

The highest order derivate is y" which is second order.

16.

(d) $\mathbb{R}$

Explanation:

$\mathbb{R}$

17.

(c) $-\frac{1}{t^2}$

Explanation:

To find $\frac{dy}{dx}$, we can use the chain rule:

$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$

First, let's find $\frac{dy}{dt}$:

$\frac{dy}{dt}=-\frac{a}{t^2}$

Next, let's find $\frac{dt}{dx}$:
$\frac{dt}{dx} = \frac{1}{a}$
Substituting these values into the chain rule equation:
$\frac{dy}{dx} = \left(-\frac{a}{t^2}\right) \cdot \left(\frac{1}{a}\right)$
Simplifying:
$\frac{dy}{dx} = -\frac{1}{t^2}$
Therefore, the correct answer is $-\frac{1}{t^2}$.

18.
(d) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$
Explanation:
Direction cosines of the line, whose direction ratios are 2, 3, -6 are:
$\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, \frac{3}{\sqrt{2^2+3^2+(-6)^2}}, \frac{-6}{\sqrt{2^2+3^2+(-6)^2}}$
or $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$
Since, line makes obtuse angle with Y-axis, then $\cos\beta < 0$
Therefore, direction cosines are $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$.

19.
(d) A is false but R is true.
Explanation:
Assertion (A) is wrong.
Since, direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.
Reason (R) is correct.
Since, the slope of the line x - 2 = 0 is ∞.
The slope of line $\sqrt{3}x$ - y - 2 = 0 is $\sqrt{3}$.
Let $m_1 = \infty$, $m_2 = \sqrt{3}$ and the angle between the given lines is θ.
$\Rightarrow \tan\theta - \left|\frac{m_2-m_1}{1+m_1 \cdot m_2}\right|$
$\Rightarrow \tan\theta = \left|\frac{\frac{m_2}{m_1}-1}{\frac{1}{m_1}+m_2}\right|$
$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$
$\Rightarrow \theta = 30^o$

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
We have, I = $\int xe^{x^2}dx$
Put x^2 = t $\Rightarrow$ 2xdx = dt
$\therefore$ I = $\frac{1}{2}\int e^t dt$
$= \frac{1}{2}e^t + C$
$= \frac{1}{2}e^{x^2} + C$

Section B

21. Let (x, y) be any point on the line containing (3, 1) and (9, 3),then the required equation is,
$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$
Expanding along R_1 , we get,
$x[1-3] - y[3-9] + 1[9-9] = 0$
$\Rightarrow -2x + 6y = 0$
$x = 3y$ which is the required equation of the line.

OR

By using the definition, on expanding the given determinant with respect to C_1

$$D = \begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$$

$\Rightarrow D = (-1)^{1+1}(2)\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + (-1)^{2+1}(1)\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} + (-1)^{3+1}(-2)\begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix}$

$\Rightarrow D = 2\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} - \begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix}$

$\Rightarrow D = 2(-6-3) - (-9+2) - 2(9+4) = -18 + 7 - 26 = -37$

22. The given integral is $\int_1^3 \frac{\cos(\log x)}{x} dx$

Let log x = t

Differentiating w.r.t. x, we get

$\frac{1}{x} dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = 3 \Rightarrow t = \log 3$

$\int_1^3 \frac{\cos(\log x)}{x} dx$

$= \int_0^{\log 3} \cos t dt \; [\because \int \cos t = \sin t]$

$= [\sin t]_0^{\log 3}$

=sin (log 3) - sin 0

$= \sin(\log 3)$

$\int_1^3 \frac{\cos(\log x)}{x} dx = \sin(\log 3)$

23. Let x be the length of an edge of the cube, V be the volume and S be the surface area at any time t.

Then, $V = x^3$ and $S = 6x^2$.

Given

$\frac{dV}{dt} = 8\text{cm}^3/\text{sec}$

$\Rightarrow \frac{d}{dt}(x^3) = 8 \Rightarrow 3x^2 \frac{dx}{dt} = 8 \Rightarrow \frac{dx}{dt} = \frac{8}{3x^2}$

Now, $S = 6x^2$

$\Rightarrow \quad \frac{dS}{dt} = 12x \frac{dx}{dt}$

$\Rightarrow \quad \frac{dS}{dt} = 12x \times \frac{8}{3x^2}$

$\Rightarrow \quad \frac{dS}{dt} = \frac{32}{x}$

$\Rightarrow \quad \left(\frac{dS}{dt}\right)_{x=12} = \frac{32}{12}\text{cm}^2/\text{sec} = \frac{8}{3}\text{cm}^2/\text{sec}$

OR

Given: $f(x) = (x + 2)e^{-x}$

$f'(x) = e^{-x} - e^{-x}(x+2)$

$= e^{-x}(1 - x - 2)$

$= -e^{-x}(x+1)$

For Critical points

$f'(x) = 0$

$\Rightarrow -e^{-x}(x + 1) = 0$

$\Rightarrow x = -1$

Clearly $\text{f}'(\text{x}) > 0$ if $\text{x} < -1$

$f'(x) < 0$ if $x > -1$

Hence f(x) increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

24. When a die is thrown, the sample space is

S = {1, 2, 3, 4, 5, 6}

$\Rightarrow$ n(S) = 6

Also, A : number is even and B : number Is red

$\therefore$ A = {2, 4, 6} and B = {1, 2, 3} and $A \cap B$ = {2}

$\Rightarrow$ n(A) = 3, n (B) = 3 and n(A∩B) = 1

Now, P(A) = $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

P(B) =$\frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and P(A∩B) = $\frac{n(A\cap B)}{n(S)} = \frac{1}{6}$

Now, P(A) × P(B) = $\frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.

25. $\because$ highest derivative is $\frac{dy}{dx}$. so order is 1.

And degree is highest power of highest derivative so degree is 2.

Section C

26. Let $I = \int \frac{x^3+1}{x^3-x} dx = \int \frac{(x+1)(x^2-x+1)}{x(x^2-1)} dx$

$= \int \frac{(x^2-x+1)}{x(x-1)} dx = \int \left[1 + \frac{1}{x(x-1)}\right] dx$

Consider, $\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$\Rightarrow \frac{1}{x(x-1)} = \frac{A(x-1)+Bx}{x(x-1)}$

$\Rightarrow 1 = A(x-1) + Bx$

On solving, we get $A = -1, B = 1$

$\Rightarrow \frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}$

$\therefore I = \int \left(1 + \frac{1}{x-1} - \frac{1}{x}\right) dx = x + \log|x-1| - \log|x| + c$

27. Given f(x) = $\begin{cases} \lambda\left(x^2+2\right), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous at x = 0

Since f(x) is continuous at x=0.Therefore, we have,

LHL = RHL = f(0).......(i)

Here, RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (4x+6)$

$= \lim_{h\to 0}[4(0+h)+6]$

$= \lim_{h\to 0}(4h+6)$

$= 4 \times 0 + 6 = 6$

From Eq(i), RHL = f(0)

$\Rightarrow \quad 2\lambda = 6 \Rightarrow \lambda = 3$

Now, given function becomes

f(x) = $\begin{cases} 3\left(x^2+2\right), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$

Now, let us check the differentiability at x = 0.

LHD = $\lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0} \frac{3\left[(0-h)^2+2\right]-3(0+2)}{-h}$

$= \lim_{h\to 0} \frac{3\left[h^2+2\right]-6}{-h} = \lim_{h\to 0}(-3h) = 0$

and RHD = $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} \frac{[4(0+h)+6]-3(0+2)}{h} = \lim_{h\to 0} \frac{4h}{h} = 4$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x = 0.

28. $I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$

$= \int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4(1-\cos^2 x)} dx$

$= \int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4 - 4\cos^2 x} dx$

$= \int_0^{\pi/2} \frac{\cos^2 x}{4-3\cos^2 x} dx$

$= \frac{-1}{3} \int_0^{\pi/2} \frac{-3\cos^2 x}{4-3\cos^2 x} dx$

$= \frac{-1}{3} \int_0^{\pi/2} \frac{4-3\cos^2 x-4}{4-3\cos^2 x} dx$

$= \frac{-1}{3} \int_0^{\pi/2} \left(1 - \frac{4}{4-3\cos^2 x}\right) dx$

$= \frac{-1}{3} \int_0^{\pi/2} 1 dx + \frac{4}{3} \int_0^{\pi/2} \frac{dx}{4-3\cos^2 x}$

$= \frac{-1}{3} [x]_0^{\pi/2} + \frac{4}{3} \int_0^{\pi/2} \frac{dx/\cos^2 x}{\frac{4}{\cos^2 x} - \frac{3\cos^2 x}{\cos^2 x}}$

$= \frac{-1}{3} \left[\frac{\pi}{2} - 0\right] + \frac{4}{3} \int_0^{\pi/2} \frac{\sec^2 x dx}{4\sec^2 x - 3}$

$= \frac{-\pi}{6} + \frac{4}{3} \int_0^{\pi/2} \frac{\sec^2 x dx}{4(1+\tan^2 x)-3} dx$

Put tan x = t

$\sec^2 x \, dx = dt$

$\therefore I = \frac{-\pi}{6} + \frac{4}{3} \int_0^{\infty} \frac{dt}{4(1+t^2)-3}$

$= \frac{-\pi}{6} + \frac{4}{3} \int_0^{\infty} \frac{dt}{4t^2+1}$

$= \frac{-\pi}{6} + \frac{1}{4} \times \frac{4}{3} \int_0^{\infty} \frac{dt}{t^2+\frac{1}{4}}$

$= \frac{-\pi}{6} + \frac{1}{3} \times \frac{1}{1/2} \left[\tan^{-1} \frac{t}{1/2}\right]_0^{\infty}$

$= \frac{-\pi}{6} + \frac{2}{3} \left[\tan^{-1} 2t\right]_0^{\infty}$

$= \frac{-\pi}{6} + \frac{2}{3} \left[\tan^{-1} \infty - \tan^{-1} 0\right]$

$= \frac{-\pi}{6} + \frac{2}{3} \left(\frac{\pi}{2} - 0\right)$

$= \frac{-\pi}{6} + \frac{\pi}{3}$

$= \pi/6$

OR

Let I = $= \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

= $\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)} dx$

By using the properties of definite integrals

= $\int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx - I$

$\Rightarrow$ 2I $= \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$

Put cos x = t for $x = \pi \Rightarrow t = -1, x = 0 \Rightarrow t = +1$ and $-\sin x dx = dt$

Therefore 2I $= \pi \int_1^{-1} \frac{-dt}{1+t^2} = \pi \int_{-1}^{1} \frac{dt}{1+t^2}$

= $\pi \left[\tan^{-1} t\right]_{-1}^{1} = \pi \left[\tan^{-1}(+1) - \tan^{-1}(-1)\right]$

= $\pi \left[\frac{\pi}{2}\right] = \frac{\pi^2}{2}$

$I = \frac{\pi^2}{4}$

29. The given differential equation is,

x dy + y dx = xy dx

$\Rightarrow$ x dy = (x - 1) y dx

$\Rightarrow \frac{1}{y} dy = \left(1 - \frac{1}{x}\right) dx$

$\Rightarrow \int \frac{1}{y} dy = \int \left(1 - \frac{1}{x}\right) dx$

$\Rightarrow$ log |y| = x - log |x| + C

$\Rightarrow$ log |y| - log |x| = x + C

$\Rightarrow$ log |xy| = x + C

$\Rightarrow$ |xyl = e^{x+C} ...(i)

It is given that y(1) = 1 i.e. y = 1 when x = l .Putting x = l and y = 1 in (i), we get

1 = $e^{1+C} \Rightarrow e^0 = e^{1+C} \Rightarrow$ C = -1

Putting C = -1 in (i), we get

$\therefore$ |xy| = e^{x-1}

$\Rightarrow$ xy = $\pm e^{x-1}$

$\Rightarrow y = \pm \frac{1}{x} e^{x-1}$

$\Rightarrow y=\frac{1}{x}e^{x-1}$ or, $y=-\frac{1}{x}e^{x-1}$

But, $y=-\frac{1}{x}e^{x-1}$ is not satisfied by y(1) = 1. Also, $y=\frac{1}{x}e^{x-1}$ is defined for all x $\neq 0$

Hence, $y=\frac{1}{x}e^{x-1}, x\in R$ - {0} is the required solution.

OR

The given differential equation is,

$xe^{\frac{y}{x}}-y+x\frac{dy}{dx}=0$

$\Rightarrow x\frac{dy}{dx}=y-xe^{\frac{y}{x}}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}}=\left(\frac{\mathrm{y}}{\mathrm{x}}\right)-\mathrm{e}^{\frac{\mathrm{y}}{\mathrm{x}}}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{f}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$

$v+x\frac{dv}{dx}=\left(\frac{vx}{x}\right)-e^{\frac{vx}{x}}$

$\Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=-\mathrm{e}^{\mathrm{v}}$

$\Rightarrow \frac{\mathrm{dv}}{\mathrm{e}^{\mathrm{v}}}=\frac{-\mathrm{dx}}{\mathrm{x}}$

Integrating both the sides we get:

$\Rightarrow \int\frac{\mathrm{dv}}{\mathrm{e}^{\mathrm{v}}}=-\int\frac{dx}{\mathrm{x}}+c$

$\Rightarrow$ -e^{-V} = -In|x| + c

Resubstituting the value of y = vx we get

$\Rightarrow -\mathrm{e}^{-\left(\frac{y}{x}\right)}$ = -ln|x| + c

Now, y (e) = 0

$\Rightarrow$-e$^{-(0)}$= -In|e| + c

$\Rightarrow$ c = 0

$\Rightarrow -\mathrm{e}^{-\left(\frac{y}{x}\right)}$ = -ln |x|

$\Rightarrow$ y = -x log (log|x|)

30. Let P be the point with position vector$\vec{p}=-\hat{\imath}-5\hat{\jmath}-10\hat{k}$ and Q be the point of intersection of the given line and the plane.

We have the line equation as

$\overrightarrow{\mathrm{r}}=(2\hat{\imath}-\hat{\jmath}+2\hat{\mathrm{k}})+\lambda(3\hat{\imath}+4\hat{\jmath}+12\hat{\mathrm{k}})$

$\therefore \overrightarrow{\mathrm{r}}=(2+3\lambda)\hat{\mathrm{i}}+(-1+4\lambda)\hat{\mathrm{j}}+(2+12\lambda)\hat{\mathrm{k}}$

Let the position vector of Q be $\overrightarrow{\mathrm{q}}$. As Q is a point on this line, for some scalar α, we have

$\Rightarrow \overrightarrow{\mathrm{q}}=(2+3\alpha)\hat{\mathrm{i}}+(-1+4\alpha)\hat{\mathrm{j}}+(2+12\alpha)\hat{\mathrm{k}}$

This point Q also lies on the given plane, which means this point satisfies the plane equation $\overrightarrow{\mathrm{r}}\cdot(\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}})=5$.

$\Rightarrow [(2+3\alpha)\hat{\mathbf{i}}+(-1+4\alpha)\hat{\mathbf{j}}+(2+12\alpha)\hat{\mathbf{k}}]\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})=5$

$\Rightarrow$ (2 + 3α)(1) + (–1 + 4α)(–1) + (2 + 12α)(1) = 5

$\Rightarrow$ 2 + 3α + 1 – 4α + 2 + 12α = 5

$\Rightarrow$ 11α + 5 = 5

$\Rightarrow$ 11α = 0

$\therefore \alpha = 0$

We have $\overrightarrow{\mathrm{q}}=(2+3\alpha)\hat{\mathrm{i}}+(-1+4\alpha)\hat{\mathrm{j}}+(2+12\alpha)\hat{\mathrm{k}}$

$\Rightarrow \overrightarrow{\mathrm{q}}=[2+3(0)]\hat{\mathrm{i}}+[-1+4(0)]\hat{\mathrm{j}}+[2+12(0)]\hat{\mathrm{k}}$

$\therefore \overrightarrow{\mathrm{q}}=2\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}}$

Using the distance formula, we have

$PQ=\sqrt{(2-(-1))^2+((-1)-(-5))^2+(2-(-10))^2}$

$\Rightarrow \mathrm{PQ}=\sqrt{3^2+4^2+12^2}$

$\Rightarrow \mathrm{PQ}=\sqrt{9+16+144}$

$\therefore \mathrm{PQ}=\sqrt{169}=13$

Thus, the required distance is 13 units.

OR

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{d} \perp \vec{a}, \vec{d} \cdot \vec{a} = 0 \quad \Rightarrow (a_1\hat{i} + a_2\hat{j} + a_3\hat{x})(\hat{i} - \hat{j}) = 0$

$\Rightarrow a_1 - a_2 = 0$ ...(i)

$\vec{d} \perp \vec{b}, \vec{a} \cdot \vec{b} = 0 \quad \Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)(3j - \hat{k}) = 0$

$\Rightarrow 3a_2 - a_2 = 0$ ...(ii)

$\vec{d}\vec{c} = 1 \quad \Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)(7\hat{i} - \hat{k}) = 1$

$\Rightarrow 7a_1 - a_3 = 1$ ...(iii)

Solving equation (i) and (ii) we get $3a_1 - a_3 = 0$ ...(iv)

Again solving equation (iii) & (iv) we get $a_1 = \frac{1}{4}$

From equation (i), $a_1 - a_2 = 0$ or $a_1 = a_2 = \frac{1}{4}$

From equation (ii), $3a_2 - a_2 = 0 \Rightarrow 3 \cdot \frac{1}{4} = a_3 \Rightarrow a_3 = \frac{3}{4}$

Hence, $\vec{d} = \frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$

31. Here,it is given that x + 2 y = 8 ...(1)

2x + y = 2 ...(2)

x - y = 1 ...(3)

Line (1) shaded area and origin lie on the same sides of x + 2 y = 8 Corresponding inequations is x + 2y $\leq$ 8 ,Now we have also

Line (2) shaded area and origin lie on the opposite side of 2x + y = 2

The corresponding inequation is 2x + y $\geq$ 2

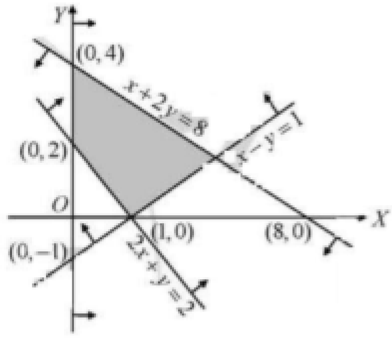
Line (3) shaded area and origin lie on the same side of x - y =1

Corresponding inequation is x - y < 1

shaded area on right side of y - axis.

Corresponding inequation is y > 0

Therefore, the linear constraints are $x \geq 0, y \geq 0, 2x + y \geq 2, x - y \leq 1$ and $x + 2y \leq 8$



Section D

32. The given equations are :

$y^2 = 16ax$ ...(1)

y = 4mx ....(2)

Equation (1) represent a parabola having centre at the origin and vertex along positive x−axis.

Equation (2) represents a straight line passing through the origin and making an angle of 45 with x−axis.

POINTS OF INTERSECTION :

Put y = 4mx in (1), we get

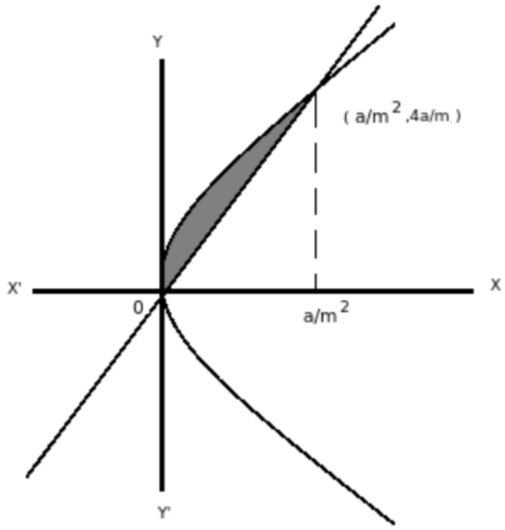
$16m^2x^2 - 16ax = 0$

$\Rightarrow 16x\,[m^2x - a] = 0$

$\Rightarrow x = 0; x = \frac{a}{m^2}$

When x = 0; y = 0

When x = $\frac{a}{m^2}$, then y = $\frac{4a}{m}$



Required area =Area under parabola - Area under line

$= 4\sqrt{a}\int_0^{a/m^2}\sqrt{x}dx - 4m\int_0^{a/m^2}xdx$

$= 4\sqrt{a} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{\frac{a}{m^2}} - \frac{4m}{2}\left[x^2\right]_0^{\frac{a}{m^2}}$

$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3}$

$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\frac{a^2}{m^3}$

Now, area = $\frac{a^2}{12}$

So, $\frac{2}{3}\frac{a^2}{m^3} = \frac{a^2}{12}$

$\Rightarrow m^3 = 8$

$\Rightarrow$ m = 2

33. According to the question, $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(i)

and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),

$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{d}) - (\vec{c} \times \vec{d}) = \vec{0}$

$\Rightarrow \quad \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = \vec{0}$

$\Rightarrow \quad \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = \vec{0}$

$[\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$

$\therefore \quad (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$

$[\because \vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$, given]

The cross-product of vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ is a zero vector, so $\vec{a} - \vec{d}$ is parallel $\vec{b} - \vec{c}$.

OR

Let $\vec{\beta} = \lambda\vec{\alpha}$ $\left[\because \vec{\beta}_1 \parallel to\, \vec{\alpha}\right]$

$\vec{\beta}_1 = \lambda(3\hat{i} - \hat{j})$

$= 3\lambda\hat{i} - \lambda\hat{j}$

$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= (2\hat{i} + \hat{j} - 3\hat{k}) - (3\lambda\hat{i} - \lambda\hat{j})$

$= (2 - 3\lambda)i + (1 + \lambda)\hat{j} - 3\hat{k}$

$\vec{\alpha}.\vec{\beta}_2 = 0$ $\left[\because \vec{\beta}_2 \perp \vec{\alpha}\right]$

$3(2 - 3\lambda) - (1 + \lambda) = 0$

$\lambda = \frac{1}{2}$

$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

34. L.H.S. = $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} -6\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} +7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} +2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} -6\begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}$$

$$+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix} + \begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$= \begin{bmatrix}5+0+16&0+0+0&10+0+24\\2+0+10&0+8+0&4+4+15\\8+0+26&0+0+0&16+0+39\end{bmatrix} - \begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix} + \begin{bmatrix}7+2&0+0&14+0\\0+0&14+2&7+0\\14+0&0+0&21+2\end{bmatrix}$$

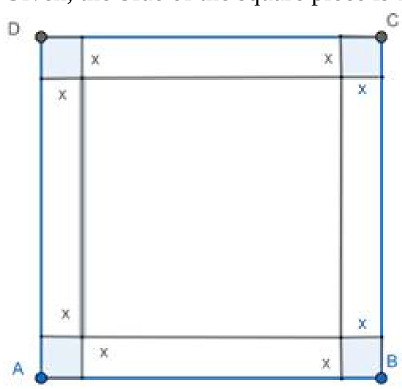
$$= \begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix} - \begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix} + \begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix} = \begin{bmatrix}21-30&0-0&34-48\\12-12&8-24&23-30\\34-48&0-0&55-78\end{bmatrix} + \begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix}$$

$$= \begin{bmatrix}-9&0&-14\\0&-16&-7\\-14&0&-23\end{bmatrix} + \begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix} = \begin{bmatrix}-9+9&0+0&-14+14\\0+0&-16+16&-7+7\\-14+14&0+0&-23+23\end{bmatrix}$$

$$= \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} = 0 \text{ (Zero matrix)}$$

= R.H.S. Proved.

35. Given, the Side of the square piece is 18 cms.



Let us consider,

'x' be the length and breadth of the piece cut from each vertex of the piece.

Side of the box now will be (18 - 2x)

The height of the newly formed box will also be 'x'.

Let the volume of the newly formed box is :

$V = (18 - 2x)^2 \times (x)$

$V = 4x^3 - 72x^2 + 324x$ ...(i)

For finding the maximum/ minimum of a given function, we can find it by differentiating it with x and then equating it to zero.

This is because if the function V(x) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (i) with respect to x:

$\frac{dV}{dx} = \frac{d}{dx}\left[4x^3 - 72x^2 + 324x\right]$

$\frac{dV}{dx} = 12x^2 - 144x + 324$ ...(ii)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (ii) to zero.

$\frac{dV}{dx} = 12x^2 - 144x + 324 = 0$

$x^2 - 12x + 27 = 0$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(27)}}{2(1)}$

$= \frac{12 \pm \sqrt{144 - 108}}{2} = \frac{12 \pm \sqrt{36}}{2}$

$x = \frac{12 \pm 6}{2}$

x = 9 or x = 3

x = 2

[as x = 9 is not a possibility, because 18 - 2x = 18 - 18 = 0]

Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (ii) with x:

$\frac{d^2V}{dx^2} = \frac{d}{dx}\left[12x^2 - 144x + 324\right]$

$\frac{d^2v}{dx^2} = 24x - 144$ ....(iv)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$\left(\frac{d^2V}{dx^2}\right)_{x=3}$ = 24(3) - 144 = 72 - 144

= -72

As $\left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$, so the function V is maximum at x = 3cm

Now substituting x = 3 in 18 - 2x, the side of the considered box:

Side = 18 - 2x = 18 - 2(3) = 18 - 6 = 12cm

Therefore side of wanted box is 12cms and height of the box is 3cms.

Now, the volume of the box is

V = $(12)^2 \times 3 = 144 \times 3 = 432cm^3$

Hence maximum volume of the box formed by cutting the given 18cms sheet is $432cm^3$ with 12cms side and 3cms height.

OR

Let r be the radius, h be the height, V be the volume and S be the total surface area of a right circular cylinder which is open at the top .

Now, given that $V = \pi r^2 h$

$\Rightarrow \quad h = \frac{V}{\pi r^2}$

We know that, total surface area S is given by

$S = 2\pi rh + \pi r^2$

[$\because$Cylinder is open at the top, therefore S= curved surface area of cylinder+area of base]

$\Rightarrow \quad S = 2\pi r\left(\frac{V}{\pi r^2}\right) + \pi r^2$

$\left[\text{put } h = \frac{V}{\pi r^2}, \text{ from Eq. (i)}\right]$

$\Rightarrow \quad S = \frac{2V}{r} + \pi r^2$

On differentiating both sides w.r.t.r, we get

$\frac{dS}{dr} = -\frac{\mathbf{2V}}{r^2} + 2\pi r$

For maxima or minima, put $\frac{dS}{dr} = 0$

$\Rightarrow \quad -\frac{2V}{r^2} + 2\pi r = 0 \Rightarrow V = \pi r^3$

$\Rightarrow \quad \pi r^2 h = \pi r^3 \quad [\because V = \pi r^2 h]$

$\Rightarrow \quad h = r$

Also, $\frac{d^2S}{dr^2} = \frac{d}{dr}\left(\frac{dS}{dr}\right) = \frac{d}{dr}\left(\frac{-\mathbf{2V}}{r^2} + 2\pi r\right)$

$\Rightarrow \quad \frac{d^2S}{dr^2} = \frac{4V}{r^3} + 2\pi$

On putting r=h, we get

$\left[\frac{d^2S}{dr^2}\right]_{r=h} = \frac{4V}{h^3} + 2\pi > 0$ as $h > 0$

Then, $\frac{d^2S}{dr^2} > 0$

Thus,S is minimum.

Hence, S is minimum, when h = r, i.e. when height of cylinder is equal to radius of the base.

Section E

36. i. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E $\cap$ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

$= \frac{P(E \cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

ii. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$= \frac{P(M \cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

iii. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P(M'/E) = \frac{P(M' \cap E)}{P(E)}$

$= \frac{P(E) - P(E \cap M)}{P(E)}$

$= \frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P(M'/E) = \frac{1}{2}$

OR

Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M) = \frac{P(E' \cap M)}{P(M)}$

$= \frac{P(M) - P(E \cap M)}{P(M)}$

$= \frac{\frac{7}{20} - \frac{1}{4}}{\frac{7}{20}} \Rightarrow P(E'/M) = \frac{2}{7}$

37. i. B = {1, 2, 3, 4, 5, 6)

R = {(x, y) : y is divisible by x}

Now, since x is divisible by x

⇒ (x, x) ∈ R

So R is reflexive

Also, here (2, 4) ∈ R, as 4 is divisible by 2

But (4, 2) ∉ R as 2 is not divisible by 4

So R is not symmetric

Now, if (x, y) ∈ R & (y, z) ∈ R

Then (x,z) ∈ R

So R is transitive

Hence R is reflexive and transitive

ii. As A has 2 elements and B has 6 elements

So, number of functions from A to B = 6^2.

iii. ∵ (1, 1) ∉ R

So R is not reflexive

Now, here (1, 2) ∈ but (2, 1) ∉ R

So R is not symmetric

Also (1, 3) ∈ R and (3, 4) ∈ R

But (1, 4) ∉ R

So R is not transitive.

OR

Number of relations

$= 2^{\text{number of element in A} \times \text{number of element in B}}$

$= 2^{2 \times 6}$

$= 2^{12}$

38. i. R.H.D. of f(x) at x = 1 = $\lim\limits_{h \to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim\limits_{h \to 0} \frac{|1+h-3|-|-2|}{h} = \lim\limits_{h \to 0} \frac{2-h-2}{h}$ = -1

ii. L.H.D. of f(x) at x = 1 = $\lim\limits_{h \to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim\limits_{h \to 0} \frac{\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]}{-h}$

$= \lim\limits_{h \to 0} \left[\frac{h^2-2h+1-6+6h+13-8}{-4h}\right]$

$= \lim\limits_{h \to 0} \left[\frac{h^2+4h}{-4h}\right]$ = -1.

iii. Since L.H.D. of f(x) at x = 1 is same as R.H.D. of f(x) at x = 1, f(x) is differentiable at x = 1.

OR

$$f(x) = \begin{cases} x-3, x \geq 3 \\ 3-x, 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$$

$[f'(x)]_{x=2} = 0 - 1 = -1$

$[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$